

Anisotropic pressure in strange quark matter under the presence of a strong magnetic field

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Abstract

Thermodynamic properties of strange quark matter in strong magnetic fields H up to 10^{20} G are considered within the MIT bag model at zero temperature implying the constraints of total baryon number conservation, charge neutrality and chemical equilibrium. The pressure anisotropy, exhibiting in the difference between the pressures along and perpendicular to the field direction, becomes essential at $H > H_{th}$, with the estimate $10^{17} < H_{th} \lesssim 10^{18}$ G. The longitudinal pressure vanishes in the critical field H_c , which can be somewhat less or larger than 10^{18} G, depending on the total baryon number density and bag pressure. As a result, the longitudinal instability occurs in strange quark matter, which precludes: (1) a significant drop in the content of s quarks, which, otherwise, could happen at $H \sim 10^{20}$ G; (2) the appearance of positrons in weak processes in a narrow interval near $H \sim 2 \cdot 10^{19}$ G (replacing electrons). The occurrence of the longitudinal instability leaves the possibility only for electrons to reach a fully polarized state, while for all quark flavors the polarization remains mild even for the fields near H_c . The anisotropic equation of state is determined under the conditions relevant to the interiors of magnetars.

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Strange quark matter, composed of deconfined u, d and s quarks, can be the true ground state of matter, as was suggested in Refs. [1–3]. There it was found that, at zero temperature and pressure, the energy per baryon in strange quark matter for a certain range of the model QCD-related parameters can be less than that for the most stable ^{56}Fe nucleus. This conjecture, if will be confirmed, would have important astrophysical implications. In particular, strange quark matter can form strange quark stars self-bound by strong interactions [4–6]. This is in contrast to an ordinary scenario, in which neutron stars are composed of hadrons (plus some admixture of leptons to ensure charge neutrality and beta equilibrium) and are bound by gravitational forces. Also, if strange quark matter is metastable at zero pressure, it can appear in the high-density core of a neutron star as a result of the deconfinement phase transition. In this case, the stability of strange quark matter is provided by the gravitational pressure from the outer hadronic layers. Then a relevant astrophysical object is a hybrid star having a quark core and the crust of hadronic matter. Moreover, strange quark matter can be potentially encountered in the form of small nuggets called "strangelets" [3, 7]. For the review on the properties of strange quark matter and its possible forms in astrophysics, one can address to Refs. [8, 9].

Another important aspect related to the physics of compact stars is that they are endowed with the magnetic field. In particular, for conventional neutron stars the magnetic field strength at the surface can reach the values in the range of 10^9 - 10^{13} G [10]. For the special classes of neutron stars such as soft γ -ray repeaters (SGRs) and anomalous X -ray pulsars (AXPs), the field strength can reach even larger values of about 10^{14} - 10^{15} G [11, 12]. These strongly magnetized objects are called magnetars [13]. It was also suggested that a magnetized hybrid star, or a magnetized strange quark star can be a real source of the SGRs or AXPs [14, 15]. The actual mechanism, by which magnetars generate such strong magnetic fields, is still under debate. Together with the dynamo amplification scenario in a magnetar with the fast rotating core [11], other possibilities such as spontaneous ordering of hadron [16], or quark [17] spins are not excluded.

Under such circumstances, the issue of interest is the impact of a strong magnetic field on thermodynamic properties of neutron star matter [18–24], or hybrid/quark star matter [25–29]. Note that in the interior of a magnetar magnetic field strength can reach even larger values compared to that at the surface. In the recent study [30], it was shown that, for either of the scenarios with a gravitationally bound neutron star or with a self-bound quark star, the field strength in the magnetar core could be as large as 10^{20} G. In such ultrastrong magnetic fields, the effects of the $O(3)$ rotational symmetry breaking by the magnetic field become important [30–33]. In particular, the longitudinal (along the magnetic field) pressure is less than the transverse (perpendicular to the magnetic field) pressure resulting in the appearance of the longitudinal instability of the star's matter if the magnetic field exceeds some critical value. The effects of the pressure anisotropy should be accounted for in the consistent study of structural and polarization properties of a strongly magnetized stellar object. As a consequence of these effects, the equation of state (EoS) of the stellar matter becomes essentially anisotropic in an ultrastrong magnetic field.

In the given research, we consider the effects of the pressure anisotropy in strange quark matter under the presence of a strong magnetic field. To study quark matter, we use a phenomenological MIT bag model [34], in which quarks of various flavors are considered as degenerate Fermi gases, confined in a finite region of space by the inward bag pressure. The MIT bag model provides simple and physically transparent framework to approach the problem and was applied earlier to the study of nonmagnetized [3, 5, 6] and magnetized [25–29] strange quark matter. Note that the effects of the pressure anisotropy were not included in the research of magnetized strange quark matter in Refs. [25–28]. Besides, as we will discuss in more detail further, the determination of the total anisotropic pressure in Ref. [29] was incomplete, where, in fact, the pure field contribution (the Maxwell term) was missed. However, just this term provides the most principal source of the pressure anisotropy in strong magnetic fields $H > H_{th}$, where $10^{17} < H_{th} \lesssim 10^{18}$ G (H_{th} is the

threshold field at which the pressure anisotropy becomes relevant). As a result, the longitudinal p_l and transverse p_t pressures interchange their roles: instead of inequality $p_t < p_l$ obtained in Ref. [29], one gets the opposite, $p_t > p_l$.

It is worthy to note at this point, that at sufficiently high density strange quark matter will be in color superconducting color-flavor-locked (CFL) state [35, 36], in which quarks of all flavors and colors near the Fermi surface are paired. Nevertheless, it is unknown which of the phases, color superconducting or normal, will be preferable at the densities closer to the density of the deconfinement phase transition. Recent perturbative QCD calculations up to the second order on the coupling constant α_s with the finite strange quark mass still do not allow nor confirm nor rule out the existence of unpaired strange quark matter, if to take into account the uncertainties in the model parameters [37]. By this reason, the study of the impact of a strong magnetic field on the thermodynamic properties of normal strange quark matter is expedient.

I. GENERAL FORMALISM

In the simplest version of the MIT bag model, quarks are considered as free fermions moving inside a finite region of space called a "bag". The effects of the confinement are accomplished by endowing the finite region with a constant energy per unit volume, the bag constant B . The bag constant B can be also interpreted as the inward pressure - the "bag pressure", needed to confine quarks inside the bag. In the MIT bag model, the bag constant is considered as a phenomenological parameter of the theory. Therefore, all relevant equations can be obtained, first, by considering the relativistic degenerate system of free fermions in an external magnetic field, and then, in order to get the equation of state of the system, by properly modifying the energy and the pressure of magnetized noninteracting fermions with account of the bag constant.

The Lagrangian density for the relativistic system of noninteracting quarks (u, d and s) and leptons (e) in an external magnetic field reads

$$\mathcal{L} = \sum_{i=u,d,s,e} \bar{\psi}_i [\gamma^\mu (i\partial_\mu - q_i A_\mu) - m_i] \psi_i - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}. \quad (1)$$

Further we will assume that the external magnetic field is directed along the z axis, and, correspondingly, choose the 4-potential in the form $A_\mu = H x_1 \delta_{\mu 2}$ ($\mu = 0, 1, 2, 3$), where H is the magnetic field strength. In Eq. (1), $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor. Analogously to Refs. [26–28], we did not take into account in Eq. (1) the quark anomalous magnetic moments which are not completely understood in the deconfined state. Also, we disregarded the electron anomalous magnetic moment, whose role, as believed, is insignificant even for the magnetic fields encountered in magnetars [38]. The quark ψ_f ($f = u, d, s$) and electron ψ_e spinors satisfy the Dirac equation

$$[\gamma^\mu (i\partial_\mu - q_i A_\mu) - m_i] \psi_i = 0, \quad i = u, d, s, e. \quad (2)$$

The energy spectrum of free relativistic fermions in an external magnetic field has the form [19]

$$\varepsilon_\nu^i = \sqrt{k_z^2 + m_i^2 + 2\nu|q_i|H}, \quad \nu = n + \frac{1}{2} - \frac{s}{2} \text{sgn}(q_i), \quad (3)$$

where $\nu = 0, 1, 2, \dots$ enumerates the Landau levels, n is the principal quantum number, $s = +1$ corresponds to a fermion with spin up, and $s = -1$ to a fermion with spin down. The lowest Landau level with $\nu = 0$ is single degenerate and other levels with $\nu > 0$ are double degenerate. For positively charged particles, the lowest Landau level is occupied by fermions with spin up, and

for negatively charged particles by fermions with spin down. As a result, each charged fermion subsystem acquires spin polarization in a magnetic field.

Further we will consider thermodynamic properties of magnetized strange quark matter at zero temperature. This approximation is quite reasonable taking into account that for the characteristic densities in the interior of a neutron star of about several times nuclear saturation density the temperature is much less than the quark chemical potentials. Concerning electrons, although finite temperature effects can be important for them, their contribution to the thermodynamic quantities is usually unessential because the electron fraction, in turn, is small. In the zero temperature case, the thermodynamic potential for an ideal gas of relativistic fermions of i th species in the external magnetic field reads [25]

$$\Omega_i = -\frac{|q_i|g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2 - \delta_{\nu,0}) \times \left\{ \mu_i k_{F,\nu}^i - \bar{m}_{i,\nu}^2 \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}, \quad (4)$$

where the factor $(2 - \delta_{\nu,0})$ takes into account the spin degeneracy of Landau levels, g_i is the remaining degeneracy factor [$g_f = 3$ for quarks (number of colors), and $g_e = 1$ for electrons], μ_i is the chemical potential, and

$$\bar{m}_{i,\nu} = \sqrt{m_i^2 + 2\nu|q_i|H}, \quad k_{F,\nu}^i = \sqrt{\mu_i^2 - \bar{m}_{i,\nu}^2}. \quad (5)$$

In Eq. (4), summation runs up to

$$\nu_{\max}^i = I\left[\frac{\mu_i^2 - m_i^2}{2|q_i|H}\right],$$

$I[\dots]$ being an integer part of the value in the brackets. The number density $\varrho_i = -(\frac{\partial \Omega_i}{\partial \mu_i})_T$ of fermions of i th species is given by

$$\varrho_i = \frac{|q_i|g_i H}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2 - \delta_{\nu,0}) k_{F,\nu}^i. \quad (6)$$

The sum in Eq. (6) can be split into two parts representing the fermion number densities with spin up and spin down. As explained earlier, the only difference between the two sums is in the term with $\nu = 0$, corresponding to spin-up fermions if they are positively charged, and to spin-down fermions, if they are negatively charged. Then the zero temperature expression for the spin polarization parameter of the i th species subsystem reads:

$$\Pi_i \equiv \frac{\varrho_i^\uparrow - \varrho_i^\downarrow}{\varrho_i} = \frac{q_i g_i H}{2\pi^2 \varrho_i} \sqrt{\mu_i^2 - m_i^2}. \quad (7)$$

In a strong enough magnetic field, when only a lowest Landau level is occupied by fermions of i th species, a full polarization occurs with $|\Pi_i| = 1$.

In order to find the chemical potentials of all fermion species (and, hence, the corresponding particle number densities), we will use the following conditions. First, the conservation of the total baryon number is implied:

$$\frac{1}{3}(\varrho_u + \varrho_d + \varrho_s) = \varrho_B, \quad (8)$$

where ϱ_B is the total baryon number density. Further, quark matter is considered to be charge neutral:

$$2\varrho_u - \varrho_d - \varrho_s - 3\varrho_{e^-} = 0. \quad (9)$$

Because of the weak processes in the quark core of a neutron star [5]

$$d \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow d + \nu_e, \quad (10)$$

$$s \rightarrow u + e^- + \bar{\nu}_e, \quad u + e^- \rightarrow s + \nu_e, \quad (11)$$

$$s + u \leftrightarrow d + u, \quad (12)$$

all fermion species are assumed to be in a chemical equilibrium with the corresponding conditions

$$\mu_d = \mu_u + \mu_{e^-}, \quad (13)$$

$$\mu_d = \mu_s. \quad (14)$$

Here we suppose that neutrinos and antineutrinos freely escape a neutron star, and, hence, their chemical potentials are set to zero. Eqs. (8), (9), (13) and (14), with account of Eq. (6), form the full set of the self-consistency equations for finding the chemical potentials μ_i of quarks and electrons.

At zero temperature, the energy density $E_i = \Omega_i + \mu_i \varrho_i$ for fermions of i th species reads

$$E_i = \frac{|q_i|g_i H}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2 - \delta_{\nu,0}) \times \left\{ \mu_i k_{F,\nu}^i + \bar{m}_{i,\nu}^2 \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}. \quad (15)$$

In the MIT bag model, the total energy density E , longitudinal p_l and transverse p_t pressures in quark matter are given by [30]

$$E = \sum_i E_i + \frac{H^2}{8\pi} + B, \quad (16)$$

$$p_l = - \sum_i \Omega_i - \frac{H^2}{8\pi} - B, \quad (17)$$

$$p_t = - \sum_i \Omega_i - HM + \frac{H^2}{8\pi} - B, \quad (18)$$

where B is the bag constant, and $M = \sum_i M_i = - \sum_i (\frac{\partial \Omega_i}{\partial H})_{\mu_i}$ is the total magnetization. At zero temperature, the magnetization of i th fermion species reads

$$M_i = \frac{|q_i|g_i}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}^i} (2 - \delta_{\nu,0}) \times \left\{ \mu_i k_{F,\nu}^i - [\bar{m}_{i,\nu}^2 + 2\nu|q_i|H] \ln \left| \frac{k_{F,\nu}^i + \mu_i}{\bar{m}_{i,\nu}} \right| \right\}. \quad (19)$$

It is seen from Eqs. (17), (18) that the magnetic field strength enters differently to the longitudinal and transverse pressures that reflects the breaking of the $O(3)$ rotational symmetry in a magnetic field. In a strong enough magnetic field, the quadratic on the magnetic field strength

term (the Maxwell term) will be dominating, leading to increasing the transverse pressure and to decreasing the longitudinal pressure. Hence, there exists a critical magnetic field H_c , at which the longitudinal pressure vanishes, resulting in the longitudinal instability of strange quark matter. Further we will find the critical magnetic field H_c for the appearance of the longitudinal instability in a quark system under the conditions relevant to the cores of magnetars. Also, we will determine the threshold magnetic field beyond which the pressure anisotropy in strange quark matter becomes significant and cannot be disregarded anymore.

Note here a principal difference between Eqs. (17), (18), used in the present study, and analogous equations in Ref. [29]. Namely, the pure field contribution (the Maxwell term) was missed in the equations of Ref. [29], and, by this reason, the authors of that work wrongly arrived at the inequality $p_l > p_t$ for the pressures, contrary to the correct result $p_l < p_t$ (for all relevant magnetic field strengths and densities, the Maxwell term is larger than the term linear on magnetization in Eq. (18), as was clarified earlier [30] and is confirmed by our calculations). Eqs. (17), (18) were obtained in Ref. [30] using the functional integration technique of a quantum field theory [39]. Also, the similar equations (after disregarding the bag pressure and small higher order terms containing M) were obtained in Refs. [32, 33] for a degenerate system of strongly interacting neutrons in an external magnetic field by applying a Fermi liquid approach [40, 41]. Moreover, the Maxwell term was also accounted for in the stress tensor of a relativistic magnetized system of free neutrons in Ref. [31], where a similar conclusion about the possibility of the collapse of a neutron star along the magnetic field (not the transverse collapse) was reached.

Note that Eqs. (16)-(18) were obtained in Ref. [30] for an abstract magnetized one-component degenerate Fermi gas. All calculations in that work were done at fixed chemical potential and zero bag pressure. We consider a scenario which is relevant in the astrophysical context of the problem, namely, we study the effects of the pressure anisotropy explicitly in magnetized three-flavor quark system (magnetized strange quark matter) under the additional constraints of the total baryon number conservation, charge neutrality and chemical equilibrium with respect to various weak processes occurring in the system. The quark species are characterized by their own chemical potentials and, together with the electron chemical potential, are determined from the constraints (8), (9), (13) and (14) at the given total baryon number density. Under such formulation of the problem, it is possible to find how the chemical composition of strange quark matter, spin polarization of various quark flavors and EoS change with the magnetic field, putting a special emphasis on the effect of the pressure anisotropy on the above dependences.

II. NUMERICAL RESULTS AND DISCUSSION

As was mentioned in Introduction, strange quark matter can be in absolutely stable state (strange quark stars), or in metastable state, which can be stabilized by high enough external pressure (hybrid stars). The valley of the absolute stability in the model parameter space is determined from the requirement that, at zero external pressure and temperature, the energy per baryon for strange quark matter should be less than that for the most stable ^{56}Fe nucleus being about 930 MeV. The maximum allowed bag pressure from the absolute stability window decreases with the magnetic field strength [42] and reaches its maximum $B \simeq 90 \text{ MeV/fm}^3$ [3] at $H = 0$. In turn, the lower bound on the bag pressure is established from the requirement that, at zero temperature and pressure, two-flavor quark matter (composed of u and d quarks) should be less stable with respect to the iron nucleus ^{56}Fe , and, hence, the energy per baryon for two-flavor quark matter should be larger than that for the nucleus ^{56}Fe [3, 8, 9]. Further, we will be interested in the astrophysical scenario, in which strange quark matter is formed in the core of a strongly magnetized neutron star, and, hence, is metastable at zero pressure. In numerical calculations, we adopt two

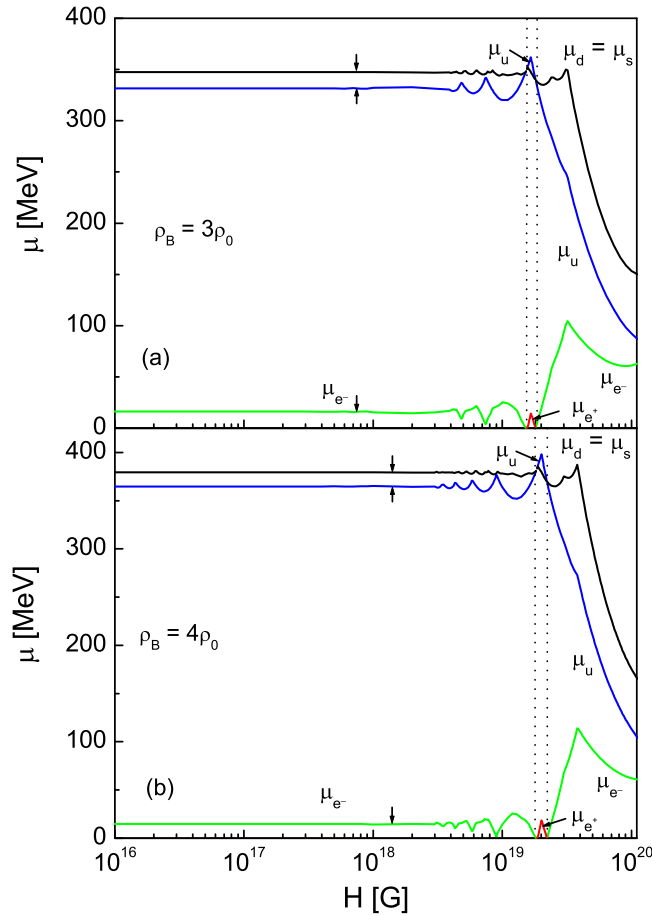


FIG. 1. (Color online) Various fermion species chemical potentials as functions of the magnetic field strength at zero temperature for the total baryon density (a) $\varrho_B = 3\varrho_0$ and (b) $\varrho_B = 4\varrho_0$. The chemical potential of positrons is shown by the red curves between the vertical dotted lines. The vertical arrows indicate the points corresponding to the critical field H_c ; see further details in the text.

values of the bag constant, $B = 100 \text{ MeV/fm}^3$ and $B = 120 \text{ MeV/fm}^3$, which are slightly larger than the upper bound on B from the absolute stability window. The core densities corresponding to these bag pressures are chosen equal to $\varrho_B = 3\varrho_0$ and $\varrho_B = 4\varrho_0$, respectively, which are, in principle, sufficient to produce deconfinement [8] ($\varrho_0 = 0.16 \text{ fm}^{-3}$ being the nuclear saturation density). For the current quark masses, we use the same values as in Refs. [25, 26, 29, 43], i.e., $m_u = m_d = 5 \text{ MeV}$, and $m_s = 150 \text{ MeV}$.

Fig. 1 shows the chemical potentials of all fermion species as functions of the magnetic field strength. It is seen that the chemical potentials of fermions, first, stay practically constant under increasing the magnetic field, with d and s quark chemical potentials being somewhat larger (on the value of $\mu_{e^-} \sim 14 - 16 \text{ MeV}$) than the u quark chemical potential. The apparent Landau oscillations of the chemical potentials appear beginning from $H \sim 3 \cdot 10^{18} - 4 \cdot 10^{18} \text{ G}$, depending on the total baryon number density. At $H \gtrsim 4 \cdot 10^{19} \text{ G}$, the quark chemical potentials decrease with the magnetic field. An interesting peculiarity occurs in a narrow interval near $H \sim 2 \cdot 10^{19} \text{ G}$, marked by the vertical dotted lines. Namely, for magnetic field strengths from that interval the u quark chemical potential is larger than the d and s quark chemical potentials, $\mu_u > \mu_d = \mu_s$. Hence, for such magnetic fields, according to Eq. (13), the electron chemical potential would be

negative, $\mu_{e^-} < 0$. If to recall the finite temperature expression for the electron number density¹, its zero temperature limit at $\mu_{e^-} < 0$ is, formally, negative, contrary to the constraint $\varrho_{e^-} \geq 0$. In fact, this means that in this interval on H electrons are missing and, hence, the weak β^- processes (10), (11) are impossible. However, for such magnetic fields, the following weak β^+ processes become allowable

$$u \rightarrow d + e^+ + \nu_e, \quad d + e^+ \rightarrow u + \bar{\nu}_e, \quad (20)$$

$$u \rightarrow s + e^+ + \nu_e, \quad s + e^+ \rightarrow u + \bar{\nu}_e. \quad (21)$$

Hence, for this specific range of the magnetic field strengths, the charge neutrality and chemical equilibrium conditions should read

$$2\varrho_u - \varrho_d - \varrho_s + 3\varrho_{e^+} = 0, \quad (22)$$

$$\mu_u = \mu_d + \mu_{e^+}, \quad \mu_d = \mu_s, \quad (23)$$

which should be solved jointly with the condition of the total baryon number conservation, Eq. (8). The quark and positron chemical potentials obtained as solutions of these equations are shown graphically in Fig. 1 as the corresponding curves between the vertical dotted lines. With increasing the core density, the width of the interval on H , where positrons appear, increases slightly as well (cf. the ranges $1.56 \cdot 10^{19} \text{ G} - 1.80 \cdot 10^{19} \text{ G}$ at $\varrho_B = 3\varrho_0$ and $1.86 \cdot 10^{19} \text{ G} - 2.21 \cdot 10^{19} \text{ G}$ at $\varrho_B = 4\varrho_0$). Thus, as a matter of principle, in strongly magnetized strange quark matter at zero temperature, subject to the total baryon number conservation, charge neutrality and chemical equilibrium conditions, positrons can appear in a certain narrow interval of the magnetic field strengths, replacing electrons. In this case, strange quark matter will have negative hadronic electric charge. Note that, according to Ref. [3], the contact of stable strange quark matter, having negative hadronic electric charge, with the ordinary matter would have the disastrous consequences for the latter, because positively charged nuclei would be attracted to strange quark matter and absorbed. However, the contact of metastable strange quark matter, having negative hadronic electric charge, with hadronic matter in the interior of a neutron star is possible, because the outer hadronic layer provides the necessary external pressure to stabilize strange quark matter in the core and cannot be completely depleted. Nevertheless, we should calculate the critical field H_c for the appearance of the longitudinal instability, which could prevent the occurrence of positrons in a certain range of magnetic field strengths with $H \gtrsim 10^{19} \text{ G}$. The meaning of the vertical arrows in Fig. 1 will be discussed later in the text.

Fig. 2 shows the abundances of various fermion species as functions of the magnetic field strength. The number densities of u and d quarks are quite close to each other for all magnetic fields under consideration. The electron number density begins quite rapidly to increase at $H \approx 2.2 \cdot 10^{16} \text{ G}$ for $\varrho_B = 3\varrho_0$ and at $H \approx 1.9 \cdot 10^{16} \text{ G}$ for $\varrho_B = 4\varrho_0$. As noted earlier, in the narrow interval near $H \sim 2 \cdot 10^{19} \text{ G}$ electrons are replaced by positrons, and beyond this interval electrons appear again with the number density increasing with H . The s quark content of strange quark matter stays practically constant till the field strength $H \approx 4.1 \cdot 10^{18} \text{ G}$ at $\varrho_B = 3\varrho_0$ and $H \approx 3.8 \cdot 10^{18} \text{ G}$ at $\varrho_B = 4\varrho_0$, beyond which the s quark number density experiences visible Landau oscillations. Then, beginning from the field strength $H \approx 3.2 \cdot 10^{19} \text{ G}$ at $\varrho_B = 3\varrho_0$ and $H \approx 3.9 \cdot 10^{19} \text{ G}$ at $\varrho_B = 4\varrho_0$, the s quark content rapidly decreases. Strange quark matter loses its strangeness and turns into two-flavor quark matter in the magnetic fields slightly larger than

¹ With account of the spin degeneracy factor $(2 - \delta_{\nu,0})$, it reads [25]:

$$\varrho_{e^-} = \frac{|q_e|H}{2\pi^2} \sum_{\nu=0}^{\infty} (2 - \delta_{\nu,0}) \int_0^{\infty} dk_z \left(\frac{1}{e^{\beta(\varepsilon_{\nu}^e - \mu_{e^-})} + 1} - \frac{1}{e^{\beta(\varepsilon_{\nu}^e + \mu_{e^-})} + 1} \right).$$

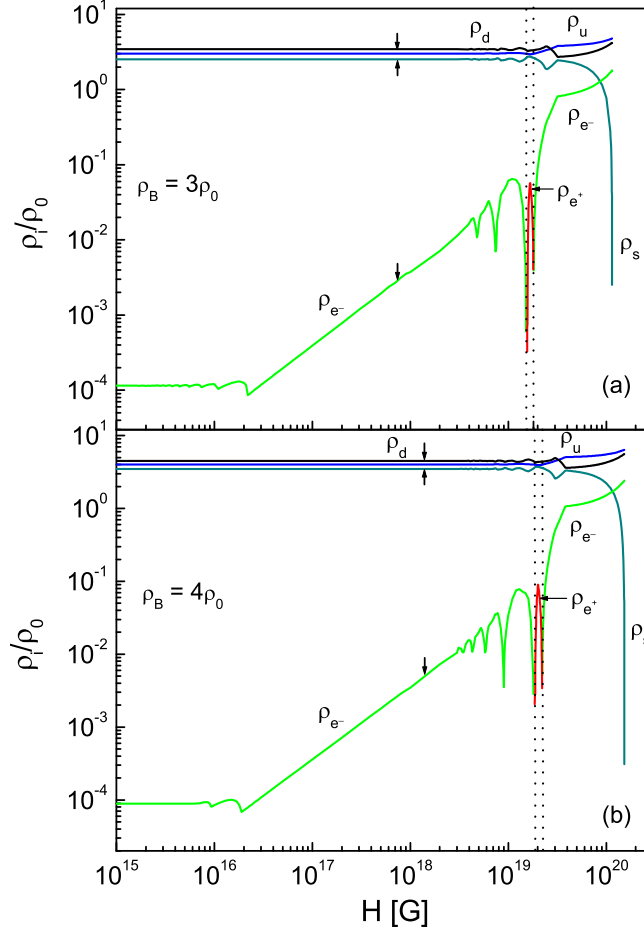


FIG. 2. (Color online) Same as in Fig. 1 but for the particle number densities ϱ_i/ϱ_0 of various fermion species. The particle number density of positrons is shown by the red curves between the vertical dotted lines.

10^{20} G. Again, we should determine the critical field H_c in order to check whether this significant drop of strangeness could happen in a strong magnetic field.

Fig. 3 shows the spin polarization parameter Π_i for various fermion species, determined according to Eq. (7), as a function of the magnetic field strength. Spin polarization of u quarks is positive while for d, s quarks and electrons it is negative. The magnitude of the spin polarization parameter Π_i increases with H till it is saturated at the respective saturation field H_s^i . At $H = H_s^i$, the corresponding i th fermion species becomes fully spin polarized. The respective values of the saturation field are: $H_s^e \approx 2.2 \cdot 10^{16}$ G for electrons, $H_s^u \approx 1.7 \cdot 10^{19}$ G for u quarks, $H_s^s \approx 2.5 \cdot 10^{19}$ G for s quarks and $H_s^d \approx 3.2 \cdot 10^{19}$ G for d quarks at $\varrho_B = 3\varrho_0$, and $H_s^e \approx 1.9 \cdot 10^{16}$ G for electrons, $H_s^u \approx 2.0 \cdot 10^{19}$ G for u quarks, $H_s^s \approx 3.1 \cdot 10^{19}$ G for s quarks and $H_s^d \approx 3.9 \cdot 10^{19}$ G for d quarks at $\varrho_B = 4\varrho_0$. Note that quite a rapid increase of the electron number density with the magnetic field (cf. Fig. 2) begins just at the saturation field H_s^e , and, hence, this increase occurs when electrons become completely spin polarized. Further oscillations in the electron number density are, in fact, caused by the Landau oscillations of the quark number densities, which influence the electron population through the charge neutrality condition. The significant drop of the strange quark content begins just at the magnetic field strength at which the d quarks become completely spin polarized. Although the s -quark current mass is larger than that for d quark, $m_s > m_d$, s quarks become fully

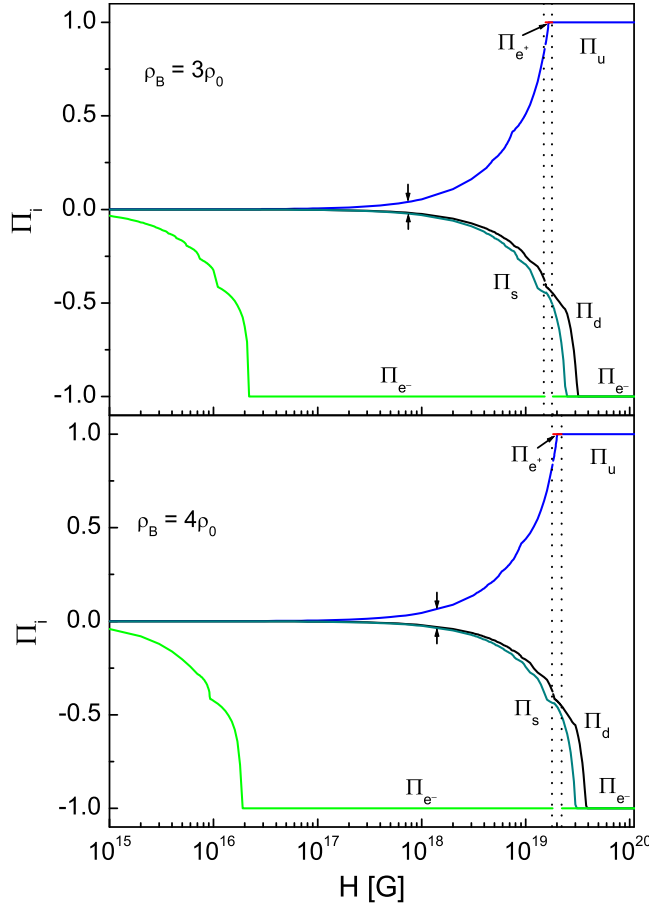


FIG. 3. (Color online) Same as in Fig. 1 but for the spin polarization parameter of various fermion species. The spin polarization parameter of positrons is shown by the red segments between the vertical dotted lines.

polarized at a smaller saturation field because their particle density is smaller than for d quarks, $\varrho_s < \varrho_d$. The spin polarization parameter of various fermion species in the magnetic field range, where positrons appear, is shown by the respective curves between the vertical dotted lines. It is seen that positrons occur already fully polarized, and u quarks become totally polarized just in this range of the magnetic field strengths. Nevertheless, as mentioned before, only after determining the critical field H_c for the appearance of the longitudinal instability, it would be possible to determine the degree of spin polarization which could be reached for each of the fermion species.

Now we present the results of calculations of the longitudinal p_l and transverse p_t pressures. Fig. 4 presents these quantities as functions of the magnetic field strength at $\varrho_B = 3\varrho_0, B = 100 \text{ MeV/fm}^3$ (solid lines) and $\varrho_B = 4\varrho_0, B = 120 \text{ MeV/fm}^3$ (dashed lines). It is seen that the transverse (ascending branches) and longitudinal (descending branches) pressures, first, stay practically constant and indistinguishable from each other. This behavior of p_t and p_l corresponds to the isotropic regime. Beyond some threshold magnetic field H_{th} , the transverse pressure p_t increases with H while the longitudinal pressure p_l decreases with it, clearly reflecting the anisotropic nature of the total pressure in strange quark matter in such strong magnetic fields (anisotropic regime). In the critical magnetic field H_c , the longitudinal pressure p_l vanishes. This happens at $H_c \approx 7.4 \cdot 10^{17} \text{ G}$ for $\varrho_B = 3\varrho_0, B = 100 \text{ MeV/fm}^3$, and at $H_c \approx 1.4 \cdot 10^{18} \text{ G}$ for $\varrho_B = 4\varrho_0, B = 120 \text{ MeV/fm}^3$. Above the critical magnetic field, the longitudinal pressure is negative leading to the longitudinal instability of strange quark matter. Therefore, the thermodynamic

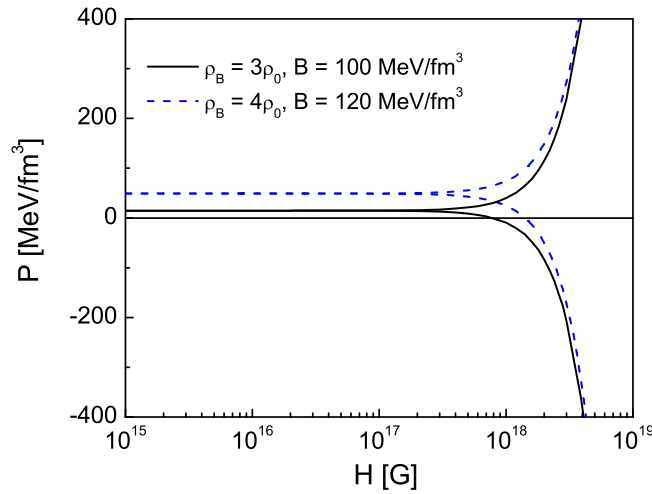


FIG. 4. (Color online) Transverse (ascending branches in the branching curves) and longitudinal (descending branches in the branching curves) pressures in magnetized strange quark matter at zero temperature as functions of the magnetic field strength for $\varrho_B = 3\varrho_0$, $B = 100 \text{ MeV/fm}^3$ (solid lines) and $\varrho_B = 4\varrho_0$, $B = 120 \text{ MeV/fm}^3$ (dashed lines).

properties of strange quark matter should be considered in the magnetic fields $H < H_c$. In fact, the critical field sets the upper bound on the magnetic field strength which can be reached in the core of a strongly magnetized hybrid star. For comparison, we present also the values of the critical field for dense neutron matter with the Skyrme BSk20 interaction at zero temperature being $H_c \approx 1.6 \cdot 10^{18} \text{ G}$ at $\varrho_B = 3\varrho_0$, and $H_c \approx 2.4 \cdot 10^{18} \text{ G}$ at $\varrho_B = 4\varrho_0$ [33].

Now, in order to see, which of the discussed already features of strange quark matter at zero temperature in a strong magnetic field are preserved before the appearance of the longitudinal instability, we show in Figs. 1-3 by the vertical arrows the respective values of the physical quantities corresponding to the critical field H_c . Let us begin with Fig. 1 for the chemical potentials of various fermion species. It is seen that the chemical potentials of quarks and electrons stay practically unchanged before the appearance of the longitudinal instability. The significant changes in the chemical potentials occur only in the fields $H > H_c$. In particular, the longitudinal instability precludes the appearance of positrons for which the fields $H \gtrsim 10^{19} \text{ G}$ are necessary.

Let us turn to Fig. 2 for the abundances of various fermion species. Till the critical field H_c , the content of quark species stays practically constant while the electron fraction remains quite small, $\varrho_e/\varrho_0 \lesssim 10^{-2}$. Also, there is no room for the significant drop of the strange quark content in strong magnetic fields $H \sim 10^{20} \text{ G}$ which is averted by the appearance of the longitudinal instability in the critical field H_c . In fact, despite the presence of strong magnetic fields $H \sim 10^{18} \text{ G}$, strange quark matter has the same fraction of s quarks as in the field-free case.

Let us now consider Fig. 3 for spin polarizations of various fermion species. It is seen that the full polarization in a strong magnetic field can be achieved only for electrons. For various quark species, the spin polarization remains quite moderate up to the critical magnetic field H_c . E.g., at $\varrho_B = 4\varrho_0$, $H = H_c$ we have $\Pi_u \approx 0.06$, $\Pi_d \approx -0.03$, $\Pi_s \approx -0.04$; at $\varrho_B = 3\varrho_0$, $H = H_c$, the quark spin polarizations are similar to these values with the maximum magnitude of the spin polarization parameter for u quarks, $\Pi_u \approx 0.04$. Therefore, the occurrence of a field-induced fully polarized state in strange quark matter is prevented by the appearance of the longitudinal instability in the critical magnetic field. The degree of spin polarization of various constituents is an important issue for determining the neutrino mean free paths in magnetized strange quark matter [44], and, hence, it is relevant for the adequate description of the neutrino transport and thermal evolution of a

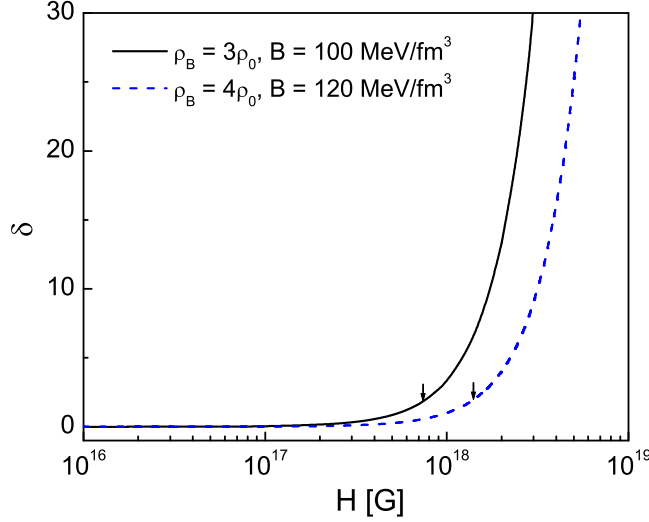


FIG. 5. (Color online) Same as in Fig. 4 but for the normalized difference $\delta = \frac{p_t - p_l}{p_0}$ between the transverse and longitudinal pressures. The vertical arrows show the maximum normalized splitting δ_c at the corresponding critical field H_c .

magnetized hybrid star.

Thus, in the anisotropic regime, the pressure anisotropy in a strong magnetic field plays an important role and should be accounted for in the description of the thermodynamic properties of strange quark matter. Let us now make the estimate of the threshold magnetic field H_{th} above which the pressure anisotropy cannot be disregarded. Fig. 5 shows the normalized difference between the transverse and longitudinal pressures

$$\delta = \frac{p_t - p_l}{p_0},$$

where p_0 is the isotropic pressure (which corresponds to the weak field limit with $p_l = p_t = p_0$), as a function of the magnetic field strength for the cases under consideration. Following Refs. [30, 32, 33], for finding the threshold field H_{th} one can use the approximate criterion $\delta \simeq 1$. Then anisotropic regime enters at $H_{th} \approx 5.5 \cdot 10^{17}$ G for $\varrho_B = 3\varrho_0, B = 100 \text{ MeV/fm}^3$, and at $H_{th} \approx 9.9 \cdot 10^{17}$ G for $\varrho_B = 4\varrho_0, B = 120 \text{ MeV/fm}^3$. For comparison [33], the threshold field for neutron matter with the BSk20 Skyrme interaction at zero temperature is $H_{th} \approx 1.2 \cdot 10^{18}$ G for $\varrho_B = 3\varrho_0$, and $H_{th} \approx 1.8 \cdot 10^{18}$ G for $\varrho_B = 4\varrho_0$. The anisotropy parameter δ reaches its maximum $\delta_c \sim 2$ in the critical field H_c , corresponding to the onset of the longitudinal instability in strange quark matter. In the anisotropic regime, a hybrid star is deformed and takes the oblate form. Thus, as follows from the previous discussions, the effects of the pressure anisotropy are important at $H_{th} < H < H_c$, and significantly influence the structural and polarization properties of the quark core in a strongly magnetized neutron star.

Fig. 6 shows the energy density E of the system without the magnetic field energy density contribution $E_f = \frac{H^2}{8\pi}$ (top panel) and with account of it (bottom panel) as a function of the magnetic field strength at zero temperature. It is seen that, due to the Landau diamagnetism, the energy density of solely magnetized strange quark matter decreases with the magnetic field. However, the overall effect of the magnetic field, with account of the pure magnetic field contribution E_f , is to increase the energy density of the system. Nevertheless, this effect of the magnetic field is, in fact, insignificant because the magnetic field is bound from above by the critical magnetic field H_c . The values of the energy density E , corresponding to the critical field H_c , are shown in Fig. 6 by the

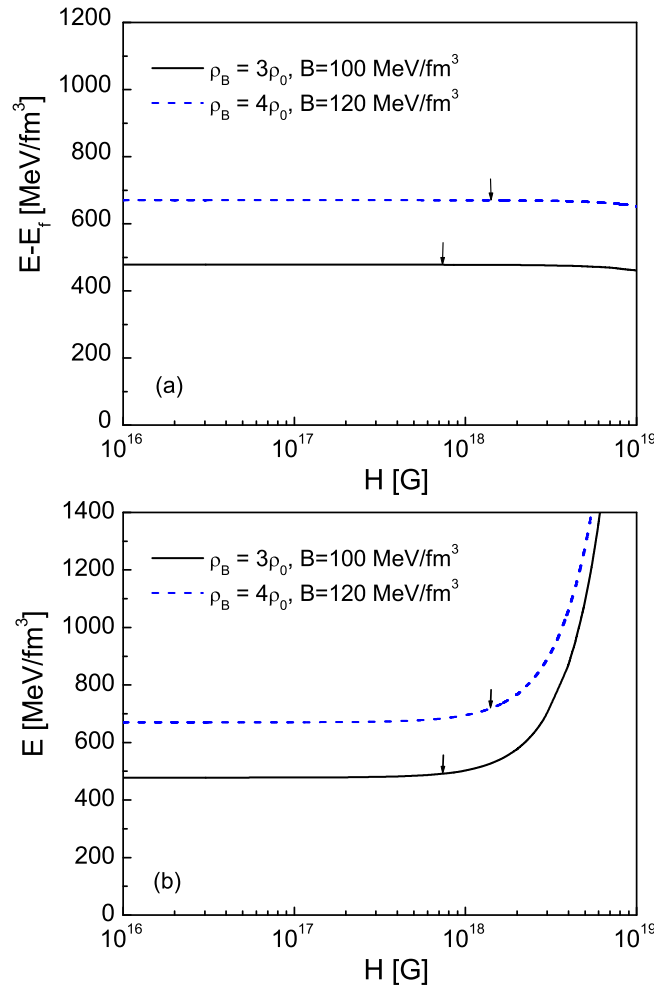


FIG. 6. (Color online) Same as in Fig. 4 but for the energy density E of the system (a) without the magnetic field energy density contribution $E_f = \frac{H^2}{8\pi}$ and (b) with account of it. The vertical arrows indicate the points corresponding to the critical field H_c .

vertical arrows.

Because of the pressure anisotropy, the equation of state of strange quark matter in a strong magnetic field is also anisotropic. Fig. 7 shows the dependence of the energy density E of the system on the transverse pressure p_t (top panel) and on the longitudinal pressure p_l (bottom panel) after excluding the dependence on H in these quantities. In particular, the anisotropic character of the pressure is reflected in the fact that the energy density is the increasing function of p_t while it decreases with p_l . This is because the dominant Maxwell term enters the transverse pressure p_t and the energy density E with positive sign while it enters the longitudinal pressure p_l with negative sign. The vertical arrows in the top panel indicate the points in these lines corresponding to the critical field H_c . In the bottom panel, the physical region corresponds to $p_l > 0$.

Note that because the EoS of strange quark matter becomes essentially anisotropic in an ultrastrong magnetic field, the usual scheme for finding the mass-radius relationship based on the Tolman-Oppenheimer-Volkoff (TOV) equations [45] for a spherically symmetric and static compact star should be revised. Instead, the corresponding relationship should be found by the self-consistent treatment of the anisotropic EoS and axisymmetric TOV equations substituting the conventional TOV equations in the case of an axisymmetric compact star.

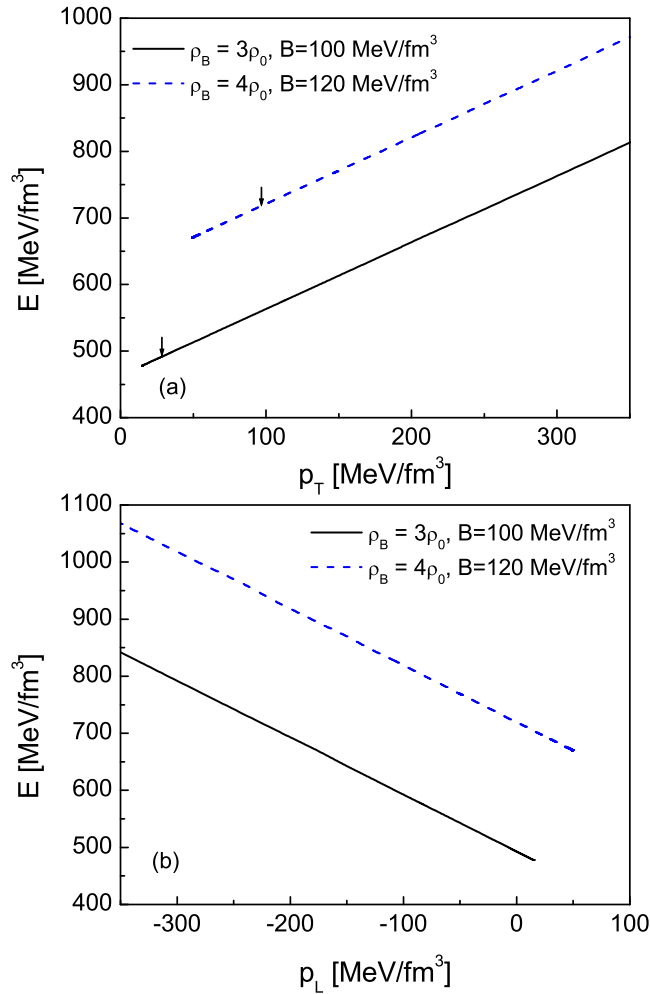


FIG. 7. (Color online) The energy density E of the system at zero temperature as a function of: (a) the transverse pressure p_t and (b) the longitudinal pressure for the cases $\varrho_B = 3\varrho_0, B = 100 \text{ MeV/fm}^3$ and $\varrho_B = 4\varrho_0, B = 120 \text{ MeV/fm}^3$. The meaning of the vertical arrows in the top panel is the same as in Fig. 6. In the bottom panel, the physical region corresponds to $p_l > 0$.

In summary, we have considered the impact of strong magnetic fields up to 10^{20} G on the thermodynamic properties of strange quark matter at zero temperature under additional constraints of total baryon number conservation, charge neutrality and chemical equilibrium with respect to various weak processes occurring in the system. The study has been done within the framework of the MIT bag model with the finite current quark masses $m_u = m_d \neq m_s$. In the numerical calculations, we have adopted two sets of the total baryon number density and bag pressure, $\varrho_B = 3\varrho_0, B = 100 \text{ MeV/fm}^3$ and $\varrho_B = 4\varrho_0, B = 120 \text{ MeV/fm}^3$. It has been found that in strong magnetic fields up to 10^{20} G some interesting features in the chemical composition and spin structure of strange quark matter could occur:

- (1) The content of strange quarks rapidly decreases in the fields somewhat larger than 10^{19} G and becomes negligible in the fields slightly exceeding 10^{20} G ;
- (2) For the magnetic field strengths in the quite narrow interval near $H \sim 2 \cdot 10^{19} \text{ G}$ the constraints of total baryon number conservation, charge neutrality and chemical equilibrium can be satisfied only if positrons appear in various weak processes in that range of the field strengths (instead of electrons);

(3) Electrons occupy only the lowest Landau level and, hence, become completely spin polarized in the magnetic fields somewhat larger than 10^{16} G; u , s and d quarks become fully polarized in the fields somewhat larger than 10^{19} G (the recitation of the quark species is in the order in which they appear fully polarized under increasing H).

Nevertheless, under such strong magnetic fields, the total pressure containing also the magnetic field contribution, becomes anisotropic, and the effects of the pressure anisotropy change most of the above conclusions. Namely, the longitudinal (along the magnetic field) pressure decreases with the magnetic field (contrary to the transverse pressure increasing with H) and vanishes in the critical field H_c resulting in the longitudinal instability of strange quark matter. The value of the critical field H_c depends on the total baryon number density of strange quark matter and the bag pressure B , and it turns out to be somewhat less or larger than 10^{18} G for the two sets of the parameters, considered in the given study. Therefore, the appearance of the longitudinal instability in strong magnetic fields beyond the critical one precludes the features (1), (2) in the chemical composition of strongly magnetized strange quark matter. Concerning the conclusion (3), only electrons can reach the state of full polarization, that is not true for quarks of all flavors, whose polarization remains mild even for magnetic fields near H_c .

The pressure anisotropy becomes relevant beyond some threshold field H_{th} . For the sets of the total baryon number density and bag constant considered in the given study, it turns out that $10^{17} < H_{th} \lesssim 10^{18}$ G. This estimate is somewhat less than that found for strongly magnetized dense neutron matter, $H_{th} \sim 10^{18}$ G [32, 33]. In strong magnetic fields $H > H_{th}$, the EoS of strange quark matter becomes essentially anisotropic. The longitudinal and transverse pressures as well as the anisotropic EoS of magnetized strange quark matter have been determined at the total baryon number densities and magnetic field strengths relevant to the interiors of magnetars.

In this work, we have studied the impact of a strong magnetic field on the thermodynamic properties of strange quark matter at zero temperature. It would be also of interest to extend this research to finite temperatures [33, 46], which can lead to a number of interesting effects, such as, e.g., an unusual behavior of the entropy of a spin polarized state [47, 48].

In conclusion, it is worthy to note that, because strong magnetic fields of about $H \sim 10^{18}$ G (RHIC), or even by order of magnitude larger (LHC), are generated in non-central high-energy heavy-ion collisions [49, 50], the effects of the pressure anisotropy should be relevant there as well. In particular, the pressure anisotropy in a strong magnetic field can contribute to the enhancement of the elliptic flow of hot nuclear matter created in a heavy-ion collision [51]. Because the conditions in high-energy heavy-ion collisions are different from those in the cores of strongly magnetized neutron stars (the absence of chemical equilibrium with respect to the weak processes, the low baryon number densities and high temperatures), the possibility for the occurrence of the strangeness suppression in a strong magnetic field for the former case needs a separate study.

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REFERENCES

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- [1] Bodmer A 1971 *Phys. Rev. D* **4** 1601
 - [2] Witten E 1984 *Phys. Rev. D* **30** 272
 - [3] Farhi E and Jaffe R L 1984 *Phys. Rev. D* **30** 2379
 - [4] Itoh N 1970 *Prog. Theor. Phys.* **44** 291

- [5] Alcock C, Farhi E and Olinto A 1986 *Astrophys. J.* **310** 261
- [6] Haensel P, Zdunik J L and Schaeffer R 1986 *Astron. Astrophys.* **160** 121
- [7] Wen X J, Feng Z Q, Li N and Peng G X 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 025011
- [8] Weber F 2005 *Prog. Part. Nucl. Phys.* **54** 193
- [9] Madsen J 1999 *Lecture Notes in Physics* **516** 162
- [10] Lyne A and Graham-Smith F 2005 *Pulsar Astronomy* (Cambridge: Cambridge University Press)
- [11] Thompson C and Duncan R C 1996 *Astrophys. J.* **473** 322
- [12] Ibrahim A I, Safi-Harb S, Swank J H, Parke W, and Zane S 2002 *Astrophys. J.* **574** L51.
- [13] Duncan R C and Thompson C 1992 *Astrophys. J.* **392** L9.
- [14] Cheng K S, and Dai Z G 2002 *Astropart. Phys.* **16** 277
- [15] Ouyed R, Leahy D, and Niebergal B 2007 *Astron. Astrophys.* **473** 357
- [16] Isayev A A, and Yang J 2004 *Phys. Rev. C* **69** 025801;
Isayev A A 2006 *Phys. Rev. C* **74** 057301
- [17] Tatsumi T 2000 *Phys. Lett. B* **489** 280
- [18] Chakrabarty S, Bandyopadhyay D, and Pal S 1997 *Phys. Rev. Lett.* **78** 2898
- [19] Broderick A, Prakash M, and Lattimer J M 2000 *Astrophys. J.* **537** 351
- [20] Cardall C, Prakash M, and Lattimer J M 2001 *Astrophys. J.* **554** 322
- [21] Perez-Garcia M A 2008 *Phys. Rev. C* **77** 065806
- [22] Isayev A A and Yang J 2009 *Phys. Rev. C* **80** 065801
- [23] Isayev A A and Yang J 2010 *J. Korean Astron. Soc.* **43** 161
- [24] Isayev A A and Yang J 2011 *Prob. Atomic Sci. Technol.* **#3** 3
- [25] Chakrabarty S 1996 *Phys. Rev. D* **54** 1306
- [26] Bandyopadhyay D, Pal S, and Chakrabarty S 1998 *J. Phys. G: Nucl. Part. Phys.* **24** 1647
- [27] Rabhi A, Pais H, Panda P K, and Providência C 2009 *J. Phys. G: Nucl. Part. Phys.* **36** 115204
- [28] Wen X J, Su S Z, Yang D H and Peng G X 2012 *Phys. Rev. D* **86** 034006
- [29] Felipe R G, Martinez A P, Rojas H P, and Orsaria M 2008 *Phys. Rev. C* **77** 015807
- [30] Ferrer E J, de la Incera V, Keith J P, Portillo I, and Springsteen P L 2010 *Phys. Rev. C* **82** 065802
- [31] Khalilov V R 2002 *Phys. Rev. D* **65** 056001
- [32] Isayev A A, and Yang J 2012 *Phys. Lett. B* **707** 163
- [33] Isayev A A, and Yang J 2011 *Phys. Rev. C* **84** 065802
- [34] Chodos A, Jaffe R L, Johnson K, Thorn C B, and Weisskopf V F 1974 *Phys. Rev. D* **9** 3471
- [35] Alford M, Rajagopal K and Wilczek F 1999 *Nucl. Phys. B* **537** 443
- [36] Lugones G and Horvath J E 2002 *Phys. Rev. D* **66** 074017
- [37] Kurkela A, Romatschke P and Vuorinen A 2010 *Phys. Rev. D* **81** 105021
- [38] Duncan R C, arXiv: astro-ph/0002442
- [39] Kapusta J I 1989 *Finite-Temperature Field Theory* (Cambridge: Cambridge University Press)
- [40] Akhiezer A I, Isayev A A, Peletminsky S V, Rekalov A P, and Yatsenko A A 1997 *JETP* **85** 1
- [41] Isayev A A and Yang J 2006 *Progress in Ferromagnetism Research*, ed V N Murray (New York: Nova Science Publishers) p 325
- [42] Paulucci L, Ferrer E J, de la Incera V, and Horvath J E 2011 *Phys. Rev. D* **83** 043009
- [43] Martinez A P, Felipe R G, Paret D M 2010 *Int. J. Mod. Phys. D* **19** 1511
- [44] Sagert I and Schaffner-Bielich J 2008 *Astron. Astrophys.* **489** 281
- [45] Tolman R C 1939 *Phys. Rev.* **55** 364;
Oppenheimer J R and Volkoff G M 1939 *Phys. Rev.* **55**, 374
- [46] Dexheimer V, Menezes D P, and Strickland M, arXiv:1210.4526.
- [47] Isayev A A 2005 *Phys. Rev. C* **72** 014313
- [48] Isayev A A 2007 *Phys. Rev. C* **76** 047305
- [49] Skokov V, Illarionov A and Toneev V 2009 *Int. J. Mod. Phys. A* **24** 5925
- [50] Deng W-T, and Huang X-G 2012 *Phys. Rev. C* **85** 044907
- [51] Tuchin K 2012 *J. Phys. G: Nucl. Part. Phys.* **39** 025010